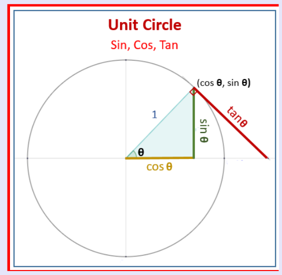
Trigonometry

Lecture 50



Feb 19-8:47 AM

Convert $5x - 4y = 20$ to a $\underbrace{\text{polar equation.}}_{r = f(\theta)}$

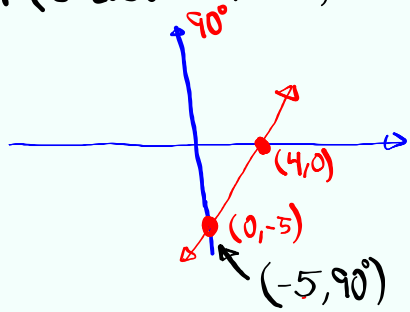
Then graph

x	y
0	-5
4	0

$$5r\cos\theta - 4r\sin\theta = 20$$

$$r(5\cos\theta - 4\sin\theta) = 20$$

$$\boxed{r = \frac{20}{5\cos\theta - 4\sin\theta}}$$



$\theta = 0°$ $\quad r = \dfrac{20}{5\cos 0 - 4\sin 0}$

$= \dfrac{20}{5} = 4$

$\theta = 90°$ $\quad r = \dfrac{20}{5\cos 90° - 4\sin 90°}$

$= \dfrac{20}{-4} = -5$

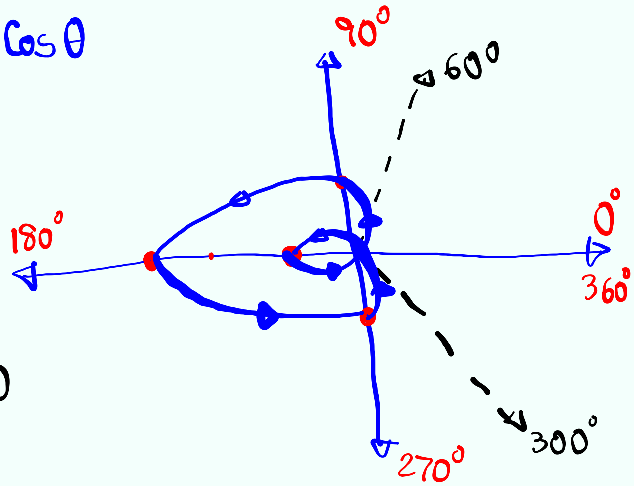
Dec 3-10:33 AM

Graph $r = 1 - 2\cos\theta$

θ	r
0°	-1
90°	1
180°	3
270°	1
360°	-1

$\to r=0$

$\to r=0$



Solve $r=0$

$$1-2\cos\theta=0$$

$$\cos\theta=\tfrac{1}{2} \quad \text{RA. } 60^\circ$$

$$\theta=60^\circ,\ \theta=300^\circ$$

Dec 3-10:39 AM

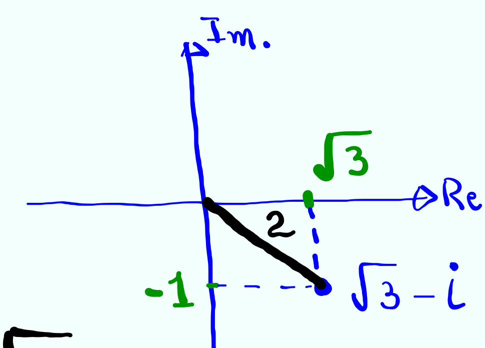
Complex number $z = a + bi$

Modulus (Abs. Value) $|z| = \sqrt{a^2+b^2}$

ex: $z = \sqrt{3} - i$

$a=\sqrt{3}$ Re

$b=-1$ Im.



$$|z| = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{4} = 2$$

Dec 3-10:45 AM

Complex number in polar form

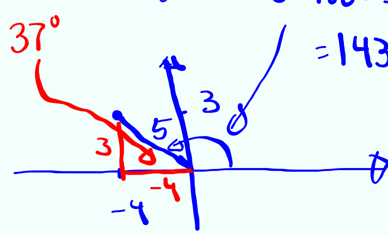
$$a + bi = r(\cos\theta + i\sin\theta)$$

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

ex: $-4 + 3i = 5(\cos 143^\circ + i \sin 143^\circ)$

$$\theta = 180^\circ - 37 = 143^\circ$$



$$r = |z| = \sqrt{(-4)^2 + 3^2} = 5$$

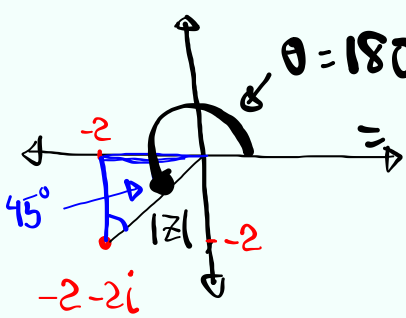
$$\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 37^\circ$$

R.A.

Dec 3-10:48 AM

$$-2 - 2i = 2\sqrt{2}(\cos 225^\circ + i \sin 225^\circ)$$

$$|z| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$



$$\theta = 180^\circ + 45^\circ = 225^\circ$$

Dec 3-10:54 AM

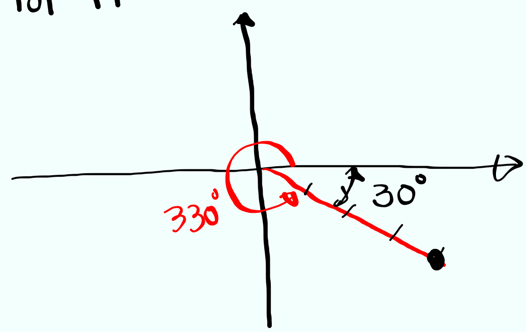
$\theta = 330^\circ$

$$Z = 4\left(\cos 330^\circ + i \sin 330^\circ\right) = 4\left[\cos 30^\circ - i \sin 30^\circ\right]$$

$|Z| = 4$

Ref. Angle

Plot it



$$= 4\left[\frac{\sqrt{3}}{2} - i \cdot \frac{1}{2}\right]$$

$$= \boxed{2\sqrt{3} - 2i}$$

Dec 3-10:57 AM

If $Z_1 = r_1\left(\cos \theta_1 + i \sin \theta_1\right)$

$Z_2 = r_2\left(\cos \theta_2 + i \sin \theta_2\right)$

$$Z_1 Z_2 = r_1 \cdot r_2\left(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right)$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)\right)$$

ex: $Z_1 = 8\left(\cos 50^\circ + i \sin 50^\circ\right)$

$Z_2 = 2\left(\cos 20^\circ + i \sin 20^\circ\right)$

$50^\circ + 20^\circ$

$$Z_1 Z_2 = 8(2)\left(\cos 70^\circ + i \sin 70^\circ\right)$$

$50^\circ - 20^\circ$

$$\frac{Z_1}{Z_2} = \frac{8}{2}\left(\cos 30^\circ + i \sin 30^\circ\right)$$

Dec 3-11:01 AM

$Z_1 = 10\ \text{CiS}\ 80°$ $\qquad$ $\text{CiS} \rightarrow \text{Cos}\ \theta + i\ \text{Sin}\ \theta$

$Z_2 = 5\ \text{Cis}\ 30°$

$Z_1 Z_2 = 10 \cdot 5\ \text{CiS}(80° + 30°) = \boxed{50\ \text{CiS}\ 110°}$

$\frac{Z_1}{Z_2} = \frac{10}{5}\ \text{CiS}(80° - 30°) = \boxed{2\ \text{CiS}\ 50°}$

$Z_1 = 4(\text{Cos}\ 100° + i\ \text{Sin}\ 100°)$ $\qquad$ $100° + 80°$

$Z_2 = 5(\text{Cos}\ 80° + i\ \text{Sin}\ 80°)$

$Z_1 Z_2 = 20(\text{Cos}\ 180° + i\ \text{Sin}\ 180°)$ $\qquad$ $100° - 80°$

$\frac{Z_1}{Z_2} = \frac{4}{5}(\text{Cos}\ 20° + i\ \text{Sin}\ 20°)$

Dec 3-11:06 AM

De Moivre's thrm:

If $Z = r(\text{Cos}\ \theta + i\ \text{Sin}\ \theta)$, then for any integer n,

$$Z^n = r^n(\text{Cos}\ n\theta + i\ \text{Sin}\ n\theta)$$

$Z = 2(\text{Cos}\ 30° + i\ \text{Sin}\ 30°)$

find $Z^3 = 2^3(\text{Cos}\ 3 \cdot 30° + i\ \text{Sin}\ 3 \cdot 30°) = 8(\text{Cos}\ 90° + i\ \text{Sin}\ 90°)$

$= 8\ \text{CiS}\ 90°$

Dec 3-11:12 AM

Given $Z = 3\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$

Find Z^5

$$3^5\left(\cos\frac{5\pi}{5} + i\sin\frac{5\pi}{5}\right)$$

$$= 243\left(\cos\pi + i\sin\pi\right)$$

$$= \boxed{243 \operatorname{cis} \pi}$$

Dec 3-11:18 AM

$Z = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Find Z^3

$$Z^3 = 4^3\left[\cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}\right]$$

$$= 64\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

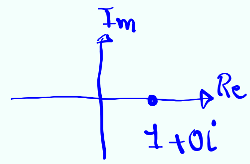
$$= 64 \operatorname{cis}\frac{\pi}{2}$$

Dec 3-11:20 AM

Solve $z^2 - 1 = 0$

$(z+1)(z-1) = 0$

$z = -1$ ✓ $\qquad z = 1$ ✓



$z^2 = 1 = 1 + 0i$

$n =$

$z^2 = 1(\cos 0° + i \sin 0°)$

$z = \sqrt{1}\left[\cos \frac{0 + k \cdot 360°}{2} + i \sin \frac{0 + k \cdot 360°}{2}\right]$

$= 1\left[\cos k \cdot 180° + i \sin k \cdot 180°\right]$

$k=0 \qquad \cos 0° + i \sin 0° = 1 + i \cdot 0 = \boxed{1}$

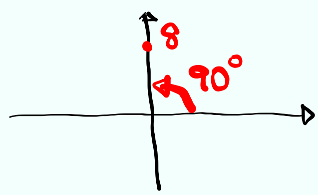
$k=1 \qquad \cos 180° + i \cdot \sin 180° = -1 + i \cdot 0 = \boxed{-1}$

Dec 3-11:24 AM

Solve $z^3 = 8i$

$= 8(i)$

$= 8(0 + 1i)$



$= 8(\cos 90° + i \sin 90°)$

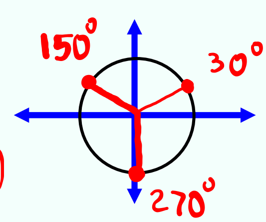
$z = \sqrt[3]{8}\left[\cos \frac{90° + k \cdot 360°}{3} + i \sin \frac{90° + k \cdot 360°}{3}\right]$

$= 2\left[\cos(30° + k \cdot 120°) + i \sin(30° + k \cdot 120°)\right]$

$k=0 \qquad 2(\cos 30° + i \sin 30°)$

$k=1 \qquad 2(\cos 150° + i \sin 150°)$

$k=2 \qquad 2(\cos 270° + i \sin 270°)$



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